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THE GENERALIZED MODEL FOR MULTI ATTRIBUTES DECISION MAKING IN A FUZZY ENVIRONMENT

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Abstract

As the nature of some decision problems has changed considerably, a serious doubts have been also raised as to the adequacy of many classical methods and their solution techniques due to the facts that much of the decision-making problems in the real world take place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely. Tthe application of classical multi criteria methods may face serious practical constraints, due to imprecision or fuzziness inherent in the information. Therefore, the application of fuzzy set theory in the field of multi criteria decision making is justified. The attitude towards fuzziness and subjectivity inherent in the process decision making has led to the new era of study applying fuzzy sets theory in the decision making area known as Fuzzy Multi Criteria Decision Making (Fuzzy-MCDM). The fuzzy-MCDM method have basically been developed along the same lines with classical MCDM method, but are designed with the help of fuzzy set theory to deal specifically with MCDM problems containing fuzzy data.

This alternative approach provides a consistent representation of qualitatively or linguistically formulated information in a way that still allows the use of precise operators and algorithms. The application of fuzzy set theory will facilitate in formulating a complex, ill-defined and subjectively perceived decision problem in an appropriate manner in such a way without make a simplification, but still in an intellectually and scientifically acceptable. The proposed method include process for identifying, measuring and aggregating criteria of alternatives to create a conceptual model for decision and evaluation in fuzzy environment.

Keywords : Fuzzy sets theory, fuzzy decision making, multi criteria, aggregation operator

1. Introduction

The most preferable situation for a multicriteria decision making problem is when all ratings of the criteria and their degree of importance are known precisely that makes possible to arrange them in a crisp ranking. However, much of the decision-making problems in the real world take place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely [1]. These situations imply that real decision problem is very complicated and thus often seem to be little suited to mathematical modeling because they are not crisply defined. Adding to this complexity is the fact that these objectives may in conflict each other and hierarchical in nature. As the nature of some decision problems has changed considerably in recent years, although the human ability to make rational decision has continuously enlarged to cope with the growing complexity, a serious doubts have been also raised as to the adequacy of many classical methods and their solution techniques [2]. Computational expertise applied to the various existing quantitative methods to solve multi criteria decision is necessary, but certainly not as sufficient condition for producing acceptable and plausible result.

One of the basic features of the human knowledge is its capability to process complex information and to communicate each other using natural language. One of the essential problems related to the information processing is the presence of imprecision or fuzziness in the available data. Ideally, the information should be precise and certain, but in the reality it is very often necessary to use information, which does not have those characteristics. The imprecision or fuzziness inherent in the information is in general they can be differentiated into intrinsic fuzziness, informartional fuzziness and relational fuzziness [3].

The expression such as tall man, high profit, reasonable economic growth and conducive environment are the examples for intrinsic fuzziness. These expressions are ill defined and the meaning perceived in these notions will depend on the decision situation and the subjective judgement based on human experiences. Informational fuzziness is due to the asymmetry between the presence of abundance information in one side and the limitation of human capability to process all information simultaneously into a single perceived criterion in other side. Relational fuzziness occurs when certain phenomena or relationships are vague due to non-dichotomy characteristics, which very frequent involved in the formulation of implicit expression.

Regarding to the causes of imprecision or fuzziness, there are a variety of sources, comprise of lack of information, abundance of information and and conflicting evidence [4,5]. Lack of information is probably could be attributed to the main source of imprecision. The imprecision may also stem from the inability to obtain the exact information due to difficulty in measuring the respective characteristic. Moreover, sometimes crisp data is technically obtainable but the cost is too high or the time required is too long and thus uses the approximation of that crisp data or even employs linguistic descriptions. To deal with the abundance information and to reduce the complexity of the decision process, the available data are generally transformed into perceivable information into a single criterion. Aggregating the available information into a perceived or condensed information and focusing attention on the criteria that seem to be the most relevant through deletion of redundant information is very useful to facilitate the overall judgement. Confronting with this kind of uncertainty, it is very clear that the higher degree of certainty cannot be achieved by gathering even more data, but by reducing the complexity through transforming the available data into appropriate perceived or condensed information.

2. Fuzzy Sets Theory

The classical or crisp set is generally defined in a such a way as to dichotomize the individuals in given universe of discourse into two groups, members an non members. There exists a sharp, unambiguous distinction between member and non-member of the set, and therefore each single element can be either belong to or not belong to the set [6,7]. To describe such classical set, one can either enumerate the element that belongs to the set, by stating condition for its membership or define the membership element by using the characteristic function, in which 1 indicates membership and 0 denotes non membership. On the contrary, a fuzzy set can be defined mathematically by assigning to each possible element in the universe of discourse a value representing its grade of membership in the fuzzy set. It is very often to express degrees of membership in sets as well as degrees of truth of the associated propositions by real numbers in the closed unit interval of [0, 1]. The capability of fuzzy sets to express gradual transition from membership to non-membership and vice versa provides a broad utility, mainly in enabling a meaningful and powerful representation of measurement uncertainties and representation of vague or ill-defined concepts express in natural language [6].

2.1 Basic Definition

Following [8], let $X = \{x\}$ denotes a collection of objects, with a generic element of X denoted by x. Then a fuzzy set A in X is a characterized by a membership function $\mu_A(x)$ which associated with each point in X a real number in the interval [0,1].

$$A = \{(x, \mu_A(x))\}, \quad \text{for } x \in X$$

Where $\mu_A(x)$ represents the grade of membership of x in A, and $\mu_A : X \to M$ is a function from X to a space M called the membership space. When M contains only two points, 0 and 1, A is non-fuzzy and its membership function becomes identical with the characteristic function of a non-fuzzy set. In the case of fuzzy set, there is a class of objects with a continuum membership grade. For the sake of simplicity, usually that M is normalized and thus can be described in closed interval of [0,1], with 0

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and 1 representing the lowest and highest grades of membership respectively [1,4]. Some basic concepts corresponding to the characteristic of fuzzy sets can be described as follows:

Normality: A fuzzy set A is normal if and only if $\mathrm{Sup}_x\ \mu_A(x) = 1$, that is, the supremum of $\mu_A(x)$ over X is unity. A fuzzy set is subnormal if it is not normal. A nonempty subnormal fuzzy set can be normalized through the division of each $\mu_A(x)$ by the factor $\mathrm{Sup}_x\ \mu_A(x)$. A fuzzy set A is empty if and only if $\mu_A(x) = 0$.

Support: The support of a fuzzy set A is a set S(A) such that $x \in S(A) \Leftrightarrow \mu_A(x) > 0$. If $\mu_A(x)$ is equal to a constant over S(A), then A is non fuzzy.

Convexity and Concavity: Let A be a fuzzy set in $X = R^n$. Then A is convex if and only if for every pair of points x, y in X, the membership function of A satisfies the inequality:

$$\mu_A(\lambda x + (1-\lambda)y) \geq \mathrm{Min}(\mu_A(x), \mu_A(y)), \text{ for } 0 \leq \lambda \leq 1.$$

Dually, A is concave if its complement A' is convex. It is easy to show that if A and B are convex, so is $A \cap B$. Dually, if A and B are concave, so is $A \cup B$.

Equality: Two fuzzy sets are equal, denoted by A=B, if and only if $\mu_A(x)=\mu_B(x)$ for all x in X.

Containment: A fuzzy set A is contained in or as a subset of a fuzzy set B, written as $A \subset B$, if and only if $\mu_A(x) \leq \mu_B(x)$.

2.2 Basic Operations of Fuzzy Sets

The basic operation used in the crisp set such as complement, intersection and union can also be applied to fuzzy set which perform precisely as the corresponding operations for crisp sets when the range of membership grades is restricted to the set {0,1}.

Intersection of Fuzzy Sets: The intersection of fuzzy set A and B is denoted by $A \cap B$ is defined as the largest fuzzy set contained in both A and B. Mathematically, the membership function of $A \cap B$ is given by:

$$\mu_{A \cap B}(x) = \mathrm{Min}(\mu_A(x), \mu_B(x)), \text{ for } x \in X$$

Where Min(a,b) = a if $a \leq b$ and Min(a,b) = b if $a > b$.

From the logical point of view, the operation of intersection of fuzzy sets corresponds to as the *connective and* that represented by the min-operator.

Union of Fuzzy Sets: The union of A and B, denoted as $A \cup B$, is defined as the smallest fuzzy set containing both A and B. The membership function of $A \cup B$ is given by:

$$\mu_{A \cup B}(x) = \mathrm{Max}(\mu_A(x), \mu_B(x)), \text{ for } x \in X$$

Where Max(a,b) = a if $a \geq b$ and Max(a,b) = b if $a < b$.

The union of A and B can be interpreted as *connective or* as modeled by the max-operator. The operation of $\vee$ and $\wedge$ are associative and distributive over one another.

Complement of Fuzzy Set: The operation of complementation corresponds to negation. The complement of fuzzy set A is denoted by A' is defined by following formula:

$$\mu_A' = 1 - \mu_A, \text{ for } x \in X$$

These standard fuzzy operations are clearly generalization of the corresponding classical set operations. However, these operations are not the only possible generalization of the classical operations [6], since there are exist a broad of class of functions whose members qualify as a fuzzy generalization. Different with the crisp operations, these standard fuzzy operations are not unique, but they can be represented by different functions in different context appropriately, due to the operations on fuzzy sets are contexts dependent.

However, due to the limitation of human being in representing the crisp value of fuzzy membership, some authors [7,9] suggested extension of the notion of a fuzzy set whose membership function itself is a fuzzy set on {0, 1}. The linguistic variable expressed by terms such as good, important or high are the common examples of fuzzy set *type B* or *type 2*. As the consequence of this extended definition, the basic operations comprise of intersection, union and complement are no longer adequate to deal with the intended operations.

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2.3 Linguistic Variable

In many situations, crisp or quantitative data are inadequate to model real-life situations, since human judgements including preferences are often vague and one cannot estimate his preference with an exact numerical value, but may be in a qualitative form [10]. One special form of fuzzy set known as linguistic variable is introduced to represent the imprecision inherent in the information or the result of verbal evaluation in which words or sentences are used in place of numbers to describe phenomena. Moreover, the linguistic terms are often the most intuitive and effective way for decision-maker to use in the evaluation process to provide more realistic and appropriate approach [11]. Each linguistic variable is expressed by linguistic terms as possible interpretation of technical figures, represented as specific fuzzy number defined in terms of a base variable [6]. A base variable is the variable in the classical sense, exemplified by any physical variable (temperature, pressure and speed) as well as numerical variable (age, interest rate and wage). The values of a linguistic variable are generated from primary terms (e.g. young and old in the case of linguistic variable Age), using various hedges (e.g. very, more or less, etc.) and connectives (AND, OR, NOT). One of the most important characteristics of linguistic variables is that the meaning of the primary terms is context-dependent, whereas the meaning of the hedges and connectives is not.

The extensions of fuzzy set definition provide more advantages, mainly for responding to the adaptability and intuitive justification or axiomatic argumentation in representing the human perception in decision making. However, these lead to the consequence, that the basic standard operations such intersection, union, and complement defined so far are inadequate for this new type of fuzzy sets. Some extensions of the basic operations are therefore required, particularly with respect to generality and adaptability of the operators. It is not possible anymore only focusing on *hard* operation, but also should capable in dealing with the *soft* operation. For example, the extensions of fuzzy set operations should also enable in carrying out other operations beyond minimum and maximum operation. Responding to this requirement, some families of fuzzy set operators have been developed and systematically presented in many related works.

3. The generalized model for fuzzy -MADM problems

Decision making problem is generally can be described as the process of searching or finding the course of actions from a given set of feasible alternatives which maximizes or satisfies certain criteria associated to the goals intended to be achieved [12]. It is concerned mainly with the question which alternative or course of action should be undertaken under a specific situation by considering many aspects, including the degree of importance of each criterion [13]. There are many variations of multi criteria decision making problems, but in general can be classified into two broad categories, namely Multi-Objective Decision Making (MODM) and Multi-Attribute Decision Making (MADM) [14,15]. The MODM concentrates on continuous decision space aiming at realization of the best solution, in which several objective functions to be achieved simultaneously and thus a MODM problem is associated to as problem of design for optimal solution through mathematical programming. On the contrary, MADM refers to making decisions in the discrete decision spaces and focuses on how to select or to rank different predetermined alternatives. Accordingly, a MADM problem can be associated to as problem of choice or ranking of the existing alternatives [2,12].

The attitude towards uncertainty and subjectivity inherent in human behavior during the process decision making has led to the new area of study applying fuzzy sets theory in the decision making area known as Fuzzy Multi Criteria Decision Making (Fuzzy-MCDM). The main feature of this approach is that the imprecision inherent in the qualitative information can be formalized by applying fuzzy sets theory. The fuzzy-MCDM methods have basically been developed along the same lines with classical MCDM method, but are designed with the help of fuzzy set theory to deal specifically with MCDM problems containing fuzzy data [3,4,7,16]. The introduction of fuzzy set theory into the field of decision making provides a consistent representation of qualitatively or linguistically formulated knowledge in a way that still allows the use of precise operators and algorithms. It also enables to represent and to process adequately the vagueness or imprecision into the formal decision

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model in such a way without make a simplification, but still in an intellectually and scientifically acceptable manner [12].

In dealing with a complex decision situation, one needs to develop a decision model as a representation of the real problem that should have similar structures and characteristics with the real problem, including in perceiving vagueness involved in the decision [17]. There are many variations on the existing fuzzy MADM method depending upon the theoretical basis used for modeling. Some authors [3,4,17,18]. addressing on the fuzzy MADM methods differentiated the family of fuzzy MADM methods into two main phases. First phase is generally known as rating process, dealing with the measurement of performance ratings or degree of satisfaction with respect to all the attributes for each alternative. The second phase, ranking of alternatives, is carried out by ordering the existing alternatives according to the resulted aggregated performance ratings obtained from the first phase. The process of ranking may result on a fuzzy as well as crisp solution to the problem, depend on the method to be applied. Zimmermann [7] classified the fuzzy methods for solving phase 2 of MADM problems into as fuzzy ranking methods and methods for solving phase 1 MADM problems and or solving both phases of MADM problems as fuzzy MADM methods.

There are a number of models or methods available to solve multi attribute decision making problem in fuzzy environment. The first approach in relating fuzzy set theory to the decision making problems was suggested by Bellman and Zadeh [1] however, the later method proposed by [Baas and Kwaakernak [19] is widely regarded as the most classical work on fuzzy MADM. The systematic review of the existing fuzzy MADM method has been done by some authors [3,4,7,17].
The specific feature which differentiates this model from the classic MADM method lies at the stage of information processing, at which the former has to deal with both quantitative as well as qualitative data. To deal with imprecision or vagueness in the decision data, fuzzy set theory will be applied to facilitate the information processing. Figure 1 describes the main phases of the development of a fuzzy MADM method.

Figure 1. The main phases of the development of a fuzzy MADM method

3.1 Identification and Formulation of the Problem

Multiple criteria decision making is characterized by the need for a complete assessment of the involved criteria before the final judgement on the alternatives can be made. Therefore, the development of a multi-criteria decision making model usually begins with the identification and formulation of the corresponding problem. The most important part of this phase is the identification of the ultimate or global goal as a basis for the formulation of the problem systematically. Unfortunately, in many real-life decision situations, the global goal is often ill-defined and thus cannot be straightforwardly identified. In most cases, the global goal could only be evaluated through an condensed indicator resulting from the aggregation of some criteria at the lower level. Considering this situation, all elements of the problem, such as the goal(s), alternatives, constraints and the surrounded environment should be precisely identified. The information should also contain a clear description of the relationship between the listed criteria and the nature of measurement of the involved criteria. Based on the obtained information, a fuzzy multi criteria decision making problem can then be formulated to represent the real decision problem.

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3.2 Construction of a Decision Model

A decision model is a simplified description of a real decision situation in such a manner that allows a systematic identification of the corresponding problem and thus provides a way for the assessment of the available alternatives. One of the suitable means for describing the complex features of a decision problem is through the construction of a decision tree that provides a more clear description about hierarchy and the logical relationship between criteria. After all the relevant criteria involved in a decision problem have been specified, their relationships with other criteria and the effect on the decision has to be described. This idea is based on the consideration that human beings due to their nature of thinking, are capable of carrying out a complex decision process step by step in a hierarchical structure, in which the global criterion is systematically derived into lower levels of sub criteria to facilitate the evaluation process. It is suggested that the listed attributes should be complete and exhaustive, containing mutually exclusive criteria and be restricted to performance attributes of the highest degree of importance [13].

It has been widely recognized that the determination of the relevant decision criteria is the most difficult task in employing the MADM method. In general, the relevant criteria to be considered may result from extensive literature survey or from a panel of experts in the problem area. In the case of the relevant criteria perhaps being identical with the goals, the direct assessment on the performance of the respective criteria can be accomplished. The judgement of the ultimate goal at the highest level of hierarchy can only be made through the aggregation of different criteria involving nominal or ordinal value in a hierarchical structure. In such a decision situation, the associated criteria should be derived from the available goals into sub criteria at a lower level representing the operationalization of the goals, until a measurable criterion is reached through which the alternatives can be assessed. However, the assessment of an alternative through the operationalization of goals may still leave some questions about how far the derived criteria can actually represent the goals. To deal with this kind of problem, the use of a hierarchical approach based on a tree-like composition is generally very useful for limiting the number of attributes, but still maintaining completeness and possible distinction between single and composite indicators [20]. Clearly, the complexity of the problem will increase according to the number of criteria to be considered, which may achieve the level beyond the human capability precisely to deal with [21].

3.3 Fuzzification of Input Variable

For the sake of information processing, the fuzzification of input variables is required to facilitate the processing of the complex information into an aggregated indicator associating with the ultimate goal. In dealing with quantitative measurements involved in the evaluation process, the crisp value corresponding to each criterion needs to be transformed into fuzzy form by the specification of its membership functions. Fuzzification function is introduced for each variable to express the measurement uncertainty of input variables, as more realistic approximation of the respective numerical values. Conversion into fuzzy data is the necessary condition if further information processing is to result in a global evaluation about the available alternatives.

There is no general membership function that applies for all quantitative measures, but the application of a particular type of membership function is often judged by the experience of a decision maker or may be even based exclusively on his or her subjective preferences. Commonly, these fuzzy sets take the form of fuzzy numbers representing the associated linguistic labels, such as in the form of a bell-shaped, triangular or trapezoidal fuzzy number. The use of a triangular membership function is appropriate under the condition that the fuzzified criterion has only one maximum value. Similarly, the trapezoidal membership function is regarded suitable for representing the measurement of a criterion expressed in the interval scale, indicating that the corresponding linguistic term may contain more that one maximum value. Concerning this issue, Rommelfanger [22] suggested that for non-technical applications, the bell-shaped membership function may be more appropriated for use. The reason for suggesting this type of membership function is due to its being based on the utility theory and normal distribution. It is suggested that the number of linguistic terms should be between five to nine, due to the fact that a human being's short term memory can only compute up to seven symbols at a time [23]. Moreover, odd linguistic terms may be used to indicate that most linguistic terms are defined symmetrically, where one term describes a middle way, representing the average value between the extremes. The interpretation of the degree of membership is straightforward: that membership value, μ_i, gives the degree to which a numerical value i satisfies the linguistic concept of a linguistic term, with 0 referring to the lowest (no) satisfaction and 1 to full satisfaction. This measurement provides us with the possibility to describe how the performance of each criterion can be described linguistically, which is very similar to the way human beings evaluate numerical values.

3.4 Aggregation Process

The basic idea of aggregation operation is to bring the information available in a large number of basic indicators or criteria into a single aggregated indicator. The aggregation process reduces the original multi criteria problem into a mono criterion problem [24] and facilitates the overall judgement about the available alternatives. The method of aggregation can take many forms, but it generally depends on the decision situation and the preference of the decision-maker. The crucial point in the aggregation process is to get empirically reliable and valid combination operators which, unfortunately, until recently was still one of the most serious problems. To arrive at the final judgement on the alternatives, the existing information should be aggregated to form a condensed indicator referring to the degree to which it satisfies the global objective. From the point of view of information processing, the aggregation process is becoming one of the most important stages in solving a multi-criteria decision-making problem, in which all the relevant criteria are considered according to their degree of importance.

One of the basic elements for the processing of fuzzy information is the employment of a fuzzy rule base inference system, comprising production rules so that its structure is similar to a decision support system. The information processing is generally expressed by a fuzzy rule base system, comprising two main components, namely **IF** portions of the statements, referred to as antecedents or premise aggregation, and **THEN** portions referring to the consequent or aggregation results [25,26]. The premise aggregation consists of a combination of all the input variables into a rule to formulate the degree to which the rule is considered appropriate for the given situation. These formulations are accounted to the most- used method in formal knowledge representation, and this method is becoming interesting, due its suitability, not only for experts, but also for ordinary people without a well-founded background in decision making. The rules of a fuzzy logic system are processed through a fuzzy rule base system, referring to cause and effect statements of a decision making process involving linguistic variables that link the input variables to output variable. It combines facts and rules about the definition and identification of the key sectors of an economy, usually by applying a rule-based approach to get a finite set of desired sectors.

Suppose that the fuzzy set A represents a certain value of an independent variable x and B represents a value of a dependent variable y, the relationship between x and y can be defined by a set of conditional statements such as on the above statement. This definition is analogous to the definition of a non-fuzzy function *f* by a table of pairs (x, *f(x)*) where x is a value of the argument of *f* and *f(x)* is the value of the function [25].

The conditional statement can be represented by a fuzzy set Z of Cartesian product $U_A \times U_B$ B with the following definition:

$$m_z(x,y) = \min [m_A(x), m_B(y)], \quad \text{for every } x \in U_A \text{ and } y \in U_B$$

Then, the grade of membership of the fuzzy value B' is given by:

$$m_{B'}(y) = \max_{x \in U_A} \{\min [m_{A'}(x), m_z(x,y)]\}$$

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The fuzzy relationship between the variables is defined by conditional statements or by acceptability statements. The criteria involved in the decision are the independent variables of the model and the final rating or the acceptability of the alternatives is the dependent one. If there are several independent variables involved, the general form of the fuzzy rule base system in the case of multi-input–single-output systems (MISO) is described by:

$$\text{IF } X_1 \text{ is } A_1 \text{ and } X_2 \text{ is } A_2, \ldots, \text{ and } X_m \text{ is } A_m \text{ THEN } Y \text{ is } B_L$$

where (IF X_1 is A_1 and X_2 is A_2,..., and X_m is A_m) are the preconditions and Y is B_L refers to the post-conditions, X_1, X_2 and X_m are input variables, Y is the output variable. A_1 is the class defined on X_1, A_m is the class defined on X_m, and B_L is class defined on Y. The antecedent or rule premise describes to what degree the rule applies, while the conclusion assigns a membership function to the output variable.

The degree of support, which takes a value of between zero to one, is used to weight the corresponding statements indicating the degree of validity of the rule. If the degree of support takes a value of 1, it means that the corresponding rule is valid without restriction. On the contrary, the value of 0 reflects the full invalidity of the respective rule. Any rule with non-zero degree of support will be taken into account. In a case where some rules lead to a particular conclusion about a different degree of conformity, the maximum of the respective degrees of membership weighted by the degree of support will determine the final result.

The choice of an appropriate aggregation operator is highly crucial, in particular when it is also intended to reflect the process of human decision making, in bringing the information contained in different criteria into one aggregated criterion which represents the total information [27,28]. The most important condition is that the aggregation operator should able to produce reasonable and acceptable results in which the descriptive and normative considerations are properly balanced. To arrive at an optimal and rational solution, it is crucial to aggregate all the involved criteria into a single indicator by considering human behavior including the possibility of trade-off between criteria. Apparently, compensation becomes the necessary condition for the existence of trade-off, in which an inferior performance with respect to one criterion can be compensated by a favorable performance with respect to one or more other criteria [29].

In many applications, the aggregation of different independent criteria is represented by a kind of generalized compensatory operator, but it becomes more complex as the number of criteria to be considered increases. If the degree of compensation takes the value 0, it reflects that there is no trade-off taken into account, and accordingly, the logical AND represented by min or product operator may be used. On the other hand, the value of 1 indicates that the decision maker is ready to make full compensation between criteria and, thus, the logical operator OR represented by max operator my be used. In real-life decision making, however, the decision maker is generally ready to make a compensation between different criteria in which the degree of compensation may take any value in the interval of [0,1] to result in the aggregated value between those two extremes

3.5 Defuzzification Process

The result of fuzzy inference about the global performance of the alternatives is described linguistically, which will certainly cause problems in the ranking process. The linguistic output resulting from the aggregation process should be converted into the numerical value which best represents the corresponding linguistic term by applying an appropriate conversion method. The accomplishment of the defuzzification process will reduce the difficulty in comparisons between linguistic output resulting from the aggregation process and thus make the final ranking of all the alternatives easier.

Since the application of fuzzy set theory in the process of decision making is aimed at mimicking the human decision and evaluation processes, the defuzzification methods used should be able to adapt to this requirement. In general, most defuzzification methods use a two-step approach, consisting of

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computing a typical value for each term in the linguistic variable followed by the search for the best compromise result through balancing out the defuzzification results [26]. A number of defuzzification methods for transforming fuzzy linguistic ratings into fuzzy numeric rating are available, but it may lead to a distinct results even when applied to the same problem. Therefore, the suitable defuzzification method should be chosen, aimed at producing plausible results according to expectations. The predominant defuzzification methods employed in a fuzzy system consist of the center of area method, the center of maximum method [30,31], the mean of maximum method [32], and the left and right assigned scores of a fuzzy number [4]. Von Altrock [26], advocated that a suitable defuzzification method is context-dependent. Comparing three different defuzzification methods, he concluded that, in general, the center of maximum method is more suitable for the application in quantitative decisions, such as budget allocation, creditworthiness or project prioritization. On the other hand, for qualitative decisions, such as credit card fraud decisions, customer segmentation or pattern recognition, the application of the mean of maximum method may be more appropriate.

3.6 Ranking of Alternatives

The results of the aggregation of a fuzzy rule-base inference system are expressed linguistically and, thus, it is still difficult for a decision maker to carry out the ranking of the available alternatives. Moreover, some fuzzy rules may yield the same inference results despite the input values being different. Confronted with this kind of situation, the ranking of the available alternatives becomes more complicated, in particular if the decision maker compares alternatives whose performances are only slightly different. The defuzzification process of the aggregated criteria will result in crisp values and this is certainly very useful in helping the decision maker to judge the global performance of each alternative.

As all aggregated indicators corresponding to each alternative have been defuzzified, the ranking of the existing alternative will pose no significant problem. The crisp number resulting from the aggregation process can be used as a basis for ranking the available alternatives. The numerical values of the variables, which are used in this calculation, are 1.0 for the best or full satisfaction and 0.0 for the worst or full dissatisfaction of the respective linguistic values. Through ordering the defuzzified values of global evaluation in descending order, the ranking of all the alternatives can be obtained. The best among all the available options then refers to the alternative having the highest value of aggregated criteria.

3.7 The practical applications of fuzzy -MADM

In the early periods of the introduction, most of the application mainly focusing on the fields of natural science, but in the recent years, fuzzy MADM methods have been also widely applied to support the decision making in various fields of study. Recently, fuzzy MADM methods have been successfully applied in the field of engineering, operations research, economic and management, quality control, medical, psychology and so on. A commercial application of fuzzy MADM for the evaluation of creditworthiness of customer of a Swiss bank has been done[27,32]. A fuzzy multi criteria analysis for performance of evaluation of bus companies has been carried out with the case of urban public transport system in Taiwan [33]. A fuzzy MADM model was used to evaluate the combine performances of ten farming systems and several hypothetical systems in Missouri [34]. The most recent application of fuzzy MADM in the field of economics has been done by Sudaryanto [33] for the identification of key sectors of an economy. They clearly recognize the usefulness of fuzzy MADM approach in dealing with the fuzziness, mainly the the advantage in adapting to the subjective evaluation involved in the decision making. The more details about the extensive practical applications of fuzzy technologies in almost all research fields are comprehensively described in the latest publication in Zimmermann [36].

4. Conclusion

Classical multi criteria decision making models have proven very effective in tackling decision problems containing crisp data, but unfortunately, they are generally inadequate for dealing with situations where imprecision and vagueness are present. The application of fuzzy set theory in the field of multi criteria decision making clearly provides additional advantage, due to it is specifically designed to enable in dealing with the decision situation in which the complexity and fuzziness are involved. Contrasting to the classical methods, the proposed approach provides some advantages, as demonstrated by its superiority in providing more realistic approach and broader perspective in interpreting the notion of key sector. It allows the consideration of all relevant criteria corresponding to the intended goals that some of them might be excluded in the conventional approach, and thus provides more reliable estimate about the decision situation. Moreover, it makes possible for the decision maker to consider trade-off between criteria leading to the rational judgement and a more satisfactory result than the existing methods so far employed and thus facilitate in resulting at the rational and acceptable compromise solution. It can be concluded that fuzzy MADM approach represent a new promising and challenging research area with the wide field of potential applications.

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